A Study of Short-Term Temporal Variations of Photon Counts Recorded by the ROSAT X-ray Satellite

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Introduction

It is usually assumed that photon events, as X-ray photons observed by a satellite, may be described by the theory of point processes (cf e. g. Matsuo, Teich and Saleh, 1983). The simplest and most common process is the homogeneous Poisson point process. Photons are usually generated in a Poisson process and in addition to that, the detection of photons is a Poisson process too. In the case of ROSAT satellite there is an additional complication. The spacecraft itself wobbles with a period of 400 seconds in order to avoid that the same parts of the sensor mosaic are constantly receiving photons from the same sources on the sky. The final result is a photon flux controlled by several Poisson processes and modulated by the spacecraft wobble. In order to study the dynamical properties of a source it is necessary to study the spectrum of variations of the Poisson parameter λ (intensity of the process), at least above the wobble frequency. For frequencies from the wobble frequency and below, a method based on the statistical analysis of the image has been proposed earlier (Liszka, 1996b).

The present study will deal with temporal variations of λ, which are much shorter than the wobble period. Of course, nothing may be said about variations shorter than the shortest λ. Another question is whether the recorded photon events really may be described by a Poisson process.

Method of analysis

The analysis is based upon the photon counts per time unit calculated from the photon event lists. If possible, photon counts per 1-second intervals are calculated. Photon counts are then low-pass filtered using a simple moving average type filter. At the beginning the Short Time Fourier Transform (STFT) was used to study the temporal variations of photon counts. However, it has been found that the use of STFT enhances the high frequency variations in comparison with the low frequency variations. Therefore, the Morlet continuous wavelet method has been used in the present study for frequency analysis.

The Morlet wavelets produce a frequency spectrum with a logarithmic frequency scale and a variable time and frequency resolution. An integral over a part of the wavelet spectrum is directly proportional to the energy confined in that part of spectrum. That is not the case for the frequency spectrum based on STFT.

Following quantities are usually considered in connection with wavelet spectrum:
- Magnitude
- Phase
- Real and Imaginary Parts
- Intermittence.

In the present study Morlet wavelets are used for analysis of temporal variations of photon counts.

The Morlet continuous wavelets method

The Morlet wavelet, being a locally periodic wavetrain, is related to windowed Fourier analysis. It is obtained by taking a complex sine wave, and by localizing it with a Gaussian (bell-shaped) envelope.
The wavelet is defined as:

$$\hat{g}(\omega) = e^{-2\pi^2(\nu-\nu_0)^2}$$

It is a complex wavelet, which can be decomposed in two parts, one for the real part, and the other for the imaginary part.

$$\begin{aligned} g_r(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \cos(2\pi\nu_0 x) \\ g_i(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \sin(2\pi\nu_0 x) \end{aligned}$$

where ν_o is a constant. The admissibility condition is verified only if $\nu_o > 0.8$.
The figure below shows these two functions.

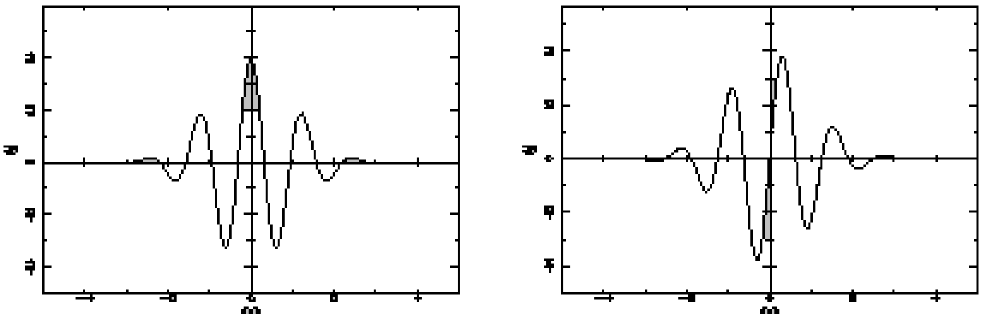


Fig. 1. Morlet's wavelet: real part at left and imaginary part at right.

The norm is the magnitude of the transform and, being related to the local energy, is of primary interest.

Morlet´s wavelet has been implemented using IDL for Windows (Research Systems Inc., 1995).

A comparison between a wavelet spectrum and an FFT spectrum

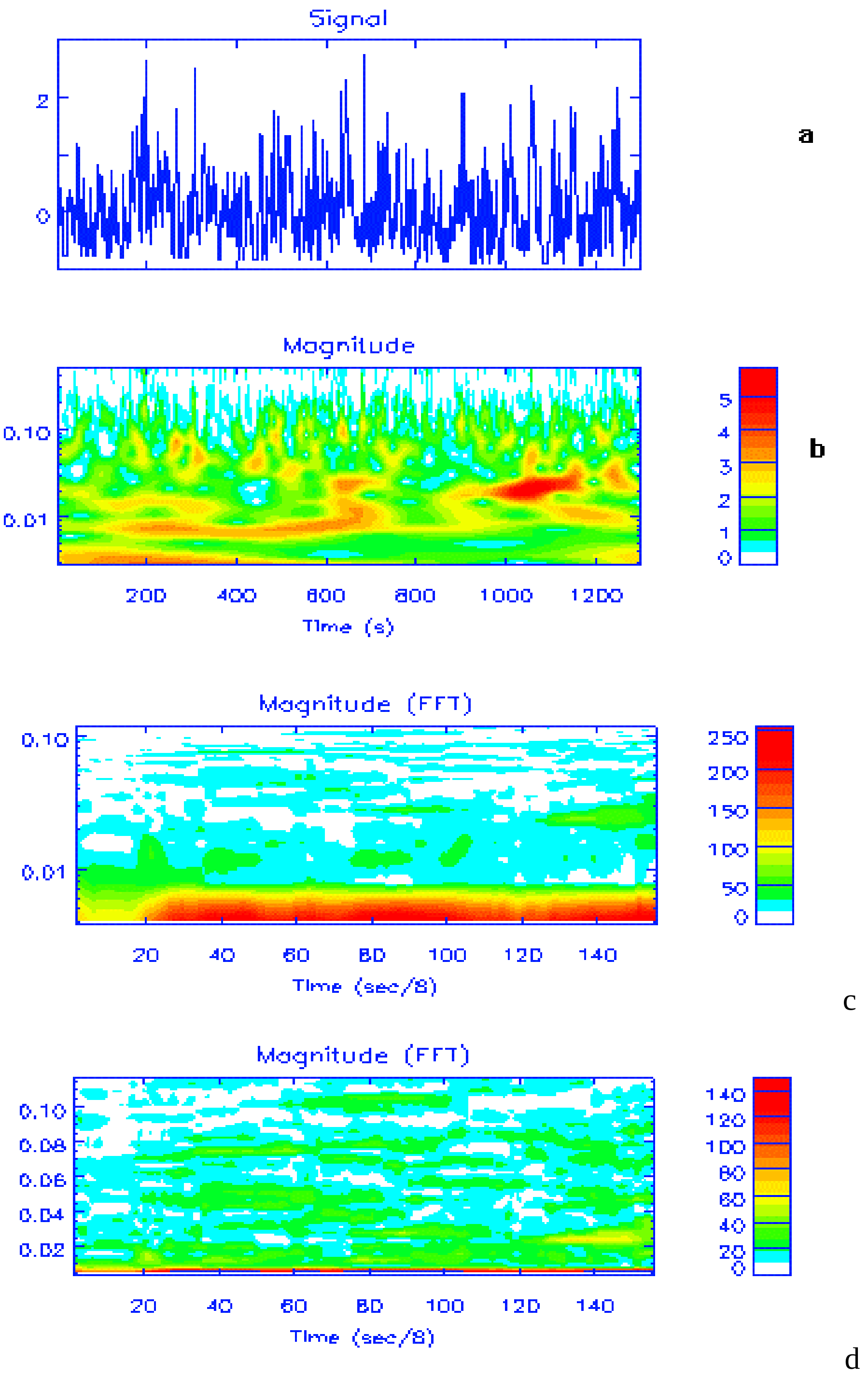


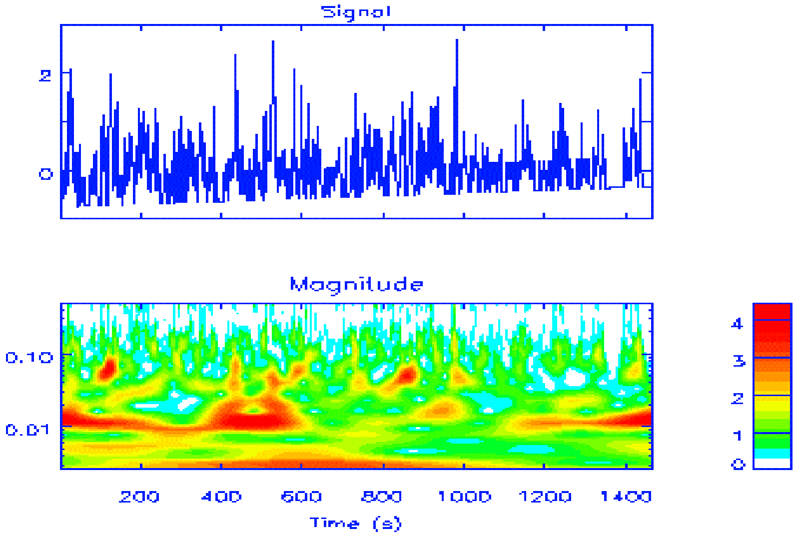
Fig. 2. An example of wavelet spectrum (b), together with the corresponding amplitude of photon counts (a) of X-ray photon counts from 3C273 during a single period of observations (energies <0.5 keV). (c) and (d) shows the corresponding FFT spectrum displayed with logarithmic, respective linear frequency scale.

An example of wavelet spectrum (b), together with the corresponding amplitude of photon counts (a), after removal of the average, is shown in the upper part of Fig. 2. In the lower part of Fig. 2 FFT spectra are shown. The spectrum (c) is displayed with a logarithmic frequency scale, while (d) is displayed using a linear frequency scale.

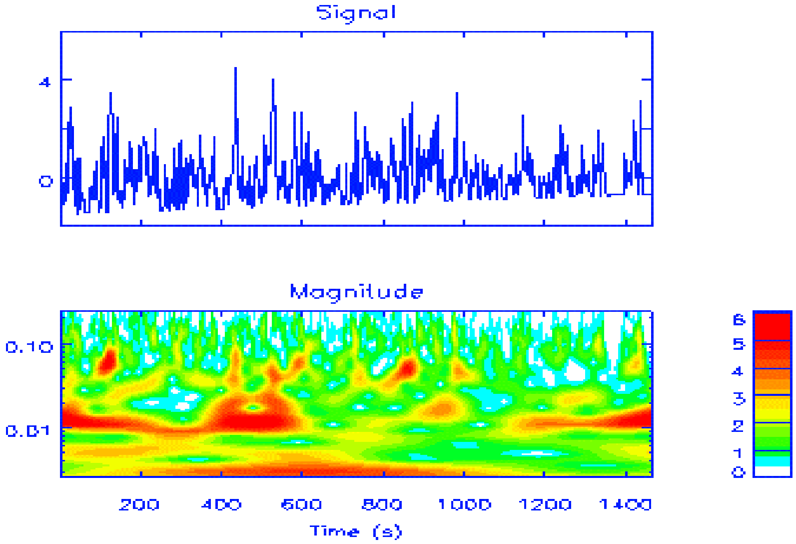
The FFT spectrum is calculated using a constant window length of 256 seconds shifted 8 seconds at a time, i. e. with a very large overlap. It appears that the structures clearly visible in the wavelet spectrum are hardly seen in the FFT spectrum, no matter if displayed on a logarithmic or linear frequency scale. The low frequency structure in (c) is due to the offset term in the spectrum, which shows up when the logarithmic frequency scale is used.

Influence of the length of counting interval

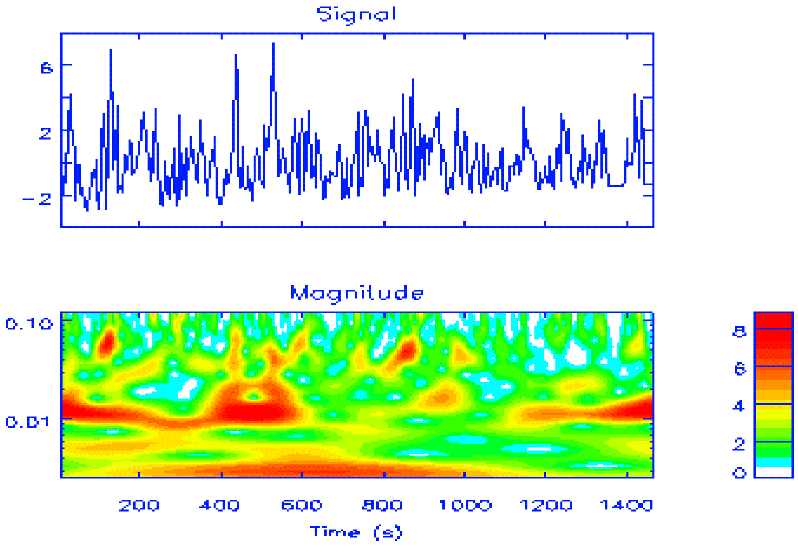
The most important problem is whether the morphology of the wavelet spectrum depends on the length of the time interval within which photons are counted. For a purely stochastic photons there would be large differences between spectra obtained with different lengths of the counting window. A sequence of counting rates and their wavelet spectra for the same period of observation is shown below.



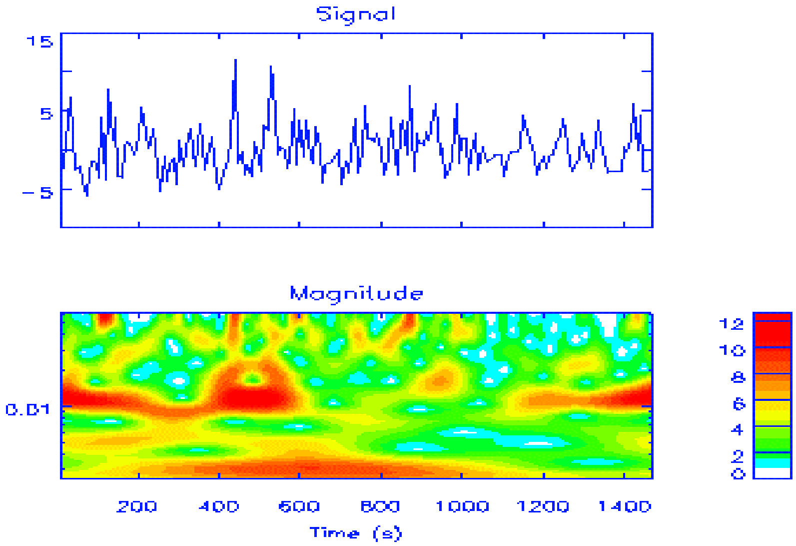
1 second interval



2 seconds interval



4 seconds interval



8 seconds interval

Fig. 3. Influence of the length of the time interval in which photons are counted on the form of the wavelet spectrum - high photon flux. NGC4051 file 557a (photon energies <0.5 keV)

It may be seen that the structure of the spectrum is extremely stable and does not change significantly with increasing length of the counting window. It should be observed that the upper frequency limit of the wavelet spectrum, of course, decreases with the increasing length of the counting window.

May the X-ray photons be fully described by a Poisson process?

There is some other evidence that X-ray photons from astronomical sources can not be fully described by a Poisson process. Klein and Arons (1990) speak about "photon bubbles" present in X-ray emission from accreting X-ray pulsars. Pacholczyk and Stoeger (1994) propose "building blocks" in the X-ray photon flux from active galactic nuclei resulting from ballistic events in the accretion disk.

There seems be a deterministic modulation of the photon series, which reflects in observed wavelet spectra of photon counts. The following experiment may be performed using the photon event data in order to test the above statement:

Let us select 5 consecutive photon events observed at instants: t_1, t_2, t_3, t_4 and t_5. Let assume that the occurrence of a photon at t_3 is conditioned by occurrences at preceding instants t_1, t_2 and following instants t_4 and t_5. Using a large population of the 5 consecutive photons it is possible to create a statistical model of the photon train using, for example, the neural network technique. That technique, being a kind of a non-linear interpolation, has been used to reconstruct uniformly sampled data, even when many data points were missing in an observed chaotic process (Liszka, 1996a).

If the photon data would follow a true Poisson process, it would be impossible to create such a model. However, it has been found, that even a simplest model consisting of a single back-propagation network, could be trained with r.m.s. errors usually less than 30%. It means that the conditional probability of a photon event at t_3, conditioned on photon events at t_1, t_2, t_4 and t_5:

$$P(t_3 \mid t_1, t_2, t_4, t_5) > 0$$

The above fact is an indication that the photon events do not follow a Poisson process. In a true Poisson process it would be impossible to predict the occurrence of an event at t_3.

Two examples of prediction of photon events at t_3 for photon data from NGC4051 observed by the ROSAT satellite are shown below. A single back-propagation neural network with 9 processing elements in the hidden layer has been used to construct a model of the photon series. Using a more complex model of hybrid type (Liszka, 1993, 1996a) it would be possible to obtain even better prediction accuracy.

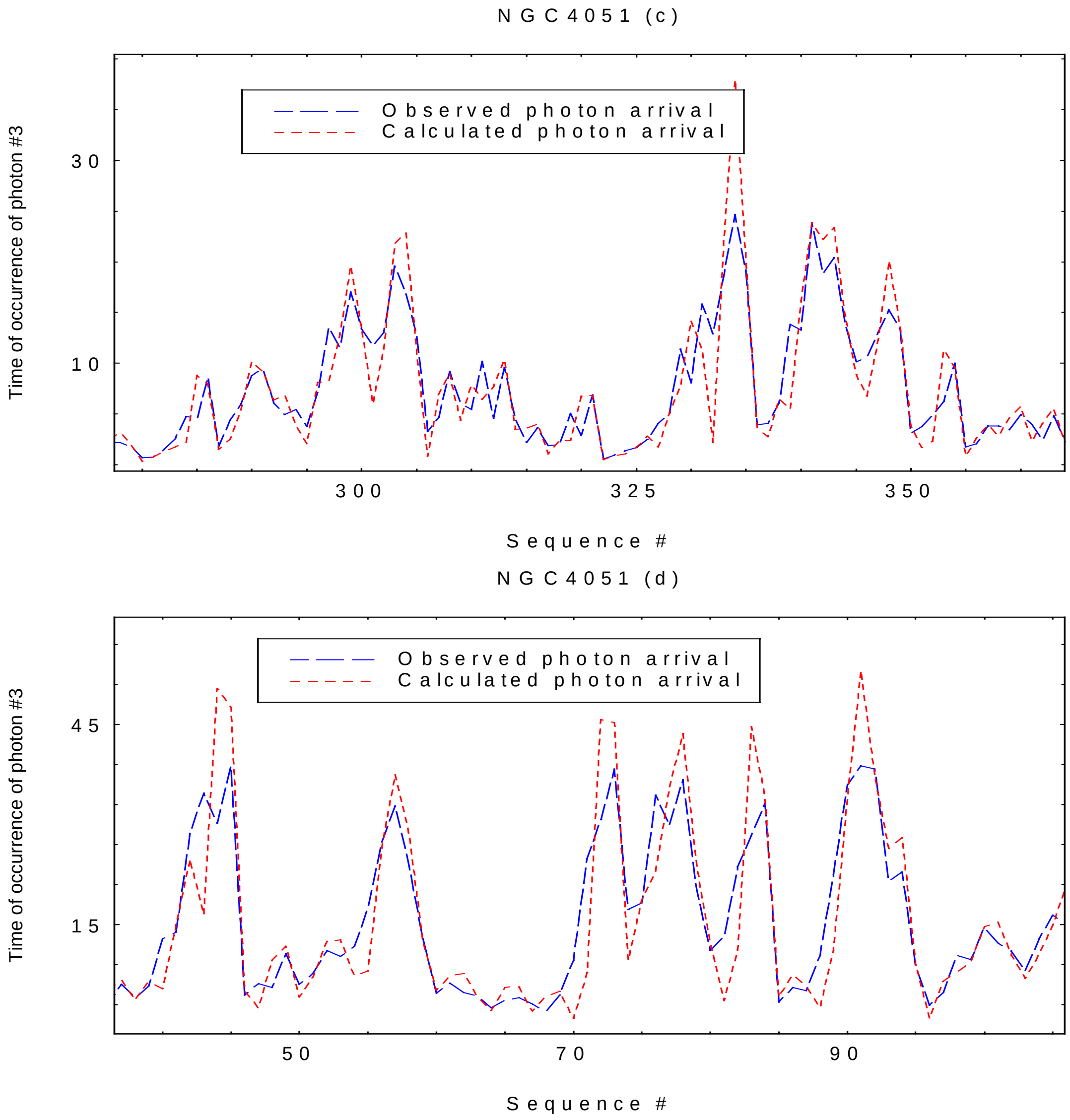


Fig. 4. Two examples of prediction of photon events at t_3 for photon data from NGC4051 observed by the ROSAT satellite. t_3 is expressed in seconds counted from t_1.

The conclusion of the above experiment is that the short-term variations of photon counts recorded by the satellite contain deterministic information, most likely corresponding to the properties of the source. Wavelet spectra seem to be a useful tool to study the short-term temporal variations in the photon counting rate.

Long-term variations of the wavelet spectrum

In order to study if the structures in the wavelet spectrum are consistent between different periods of observation of the same time sequence and if there are some differences between different sources. Each period of observation from ROSAT, which contains continuous photon data, is usually 1000 - 2000 seconds long. Average wavelet spectra were calculated for all periods of observation in each sequence. Two sequences were selected:

1. NGC5548 file wp150071 starting July 16, 1990 at 23:29:07 UT
The sequence contains 11 continuous periods of observation covering 6.5 days.
Results for the sequence are shown in Fig. 6. The upper part of Fig. 6 shows the normalized measured photon counts together with the results of principal component analysis according to the earlier work (Liszka, 1996) corresponding to the true temporal variations of photon counts. The lower part of Fig. 6 shows the long-term temporal variations of the average wavelet spectrum during the entire sequence. The temporal variations of the spectrum were derived using the 3-dimensional interpolation routine of MATLAB. The vertical axis shows the frequency channels of the wavelet spectrum. Corresponding frequencies in Hz are shown in Fig. 5. The highest spectral channel (#30) corresponds to 0.5 Hz.

2. NGC4051 file wp700557 starting November 16, 1991 at 00:39:51 UT
The sequence consists of 14 continuous periods of observation covering 21 hours.
The results for that sequence are shown in Fig. 7. The upper part of the figure shows the normalized counting rates for photon energy above 0.5 keV. It is interesting that during that sequence, lasting only 21 hours, the ratio between the highest and the lowest counting rate is about 20. Two X-ray bursts at the beginning of the sequence show approximately the same counting rates, but significantly different frequency spectrum. It may be seen in the lower part of Fig. 7. The vertical scale of the 3-D frequency spectrum shows spectral channels, corresponding to frequencies in Hz according to the conversion diagram of Fig. 5.

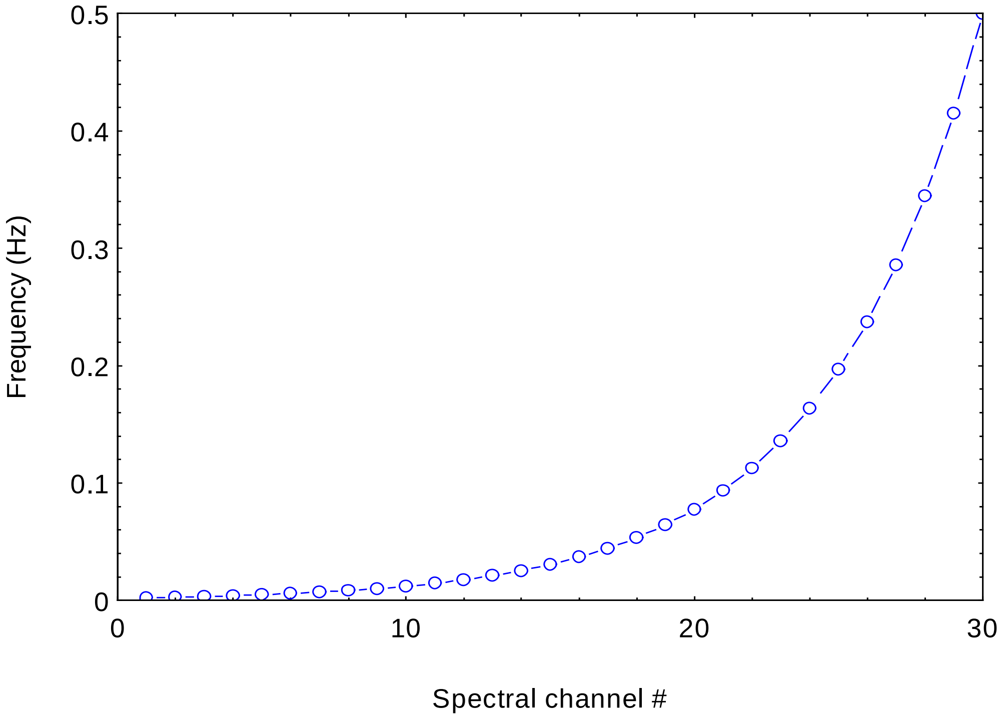


Fig. 5. Relation between the spectral channels and frequencies in Hz.

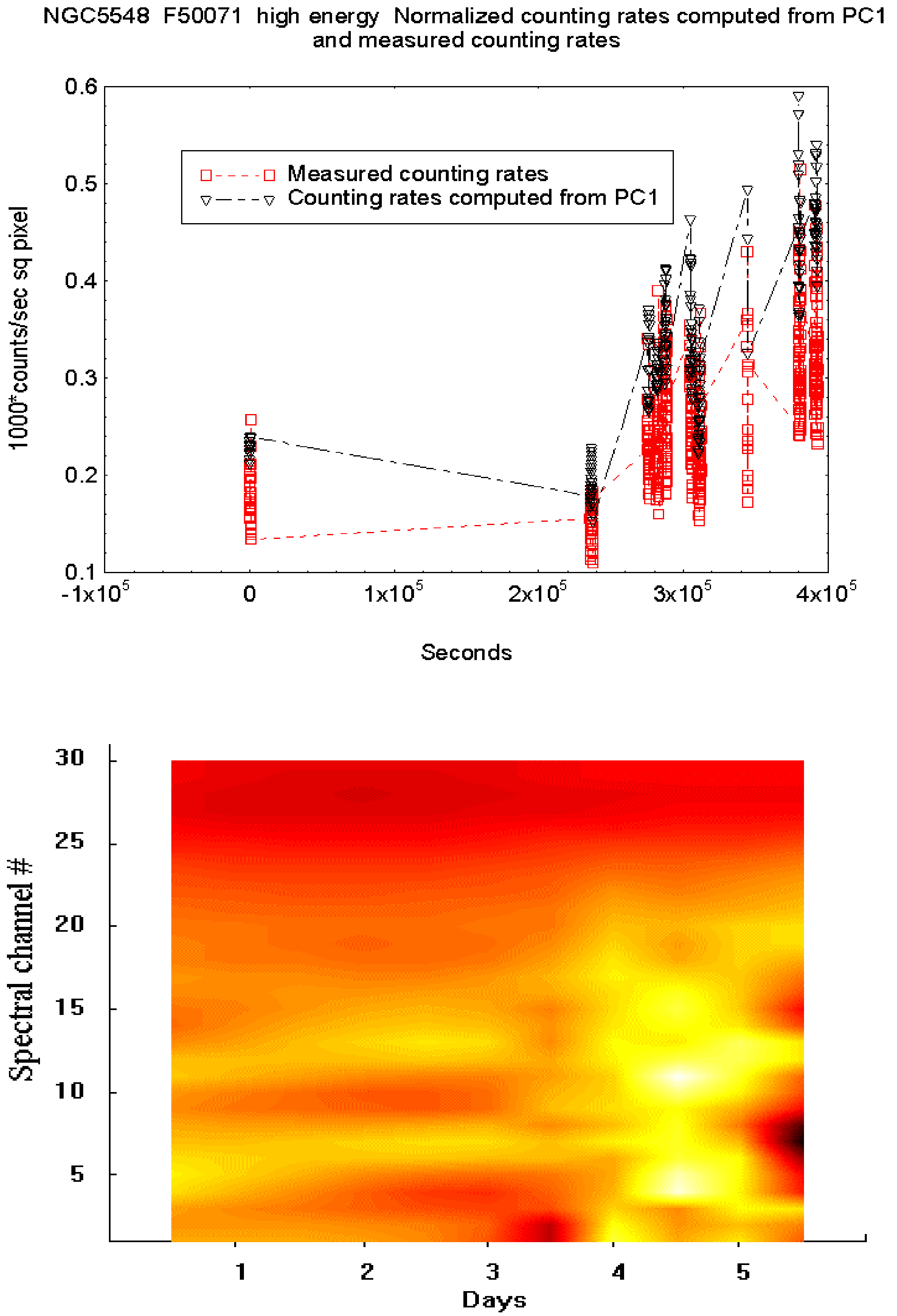


Fig. 6. NGC5548. Normalized measured counting rates for photons >0.5 keV together with rates computed from PC1 (upper diagram) and the 3-D frequency spectrum of counting rate variations during the same sequence.

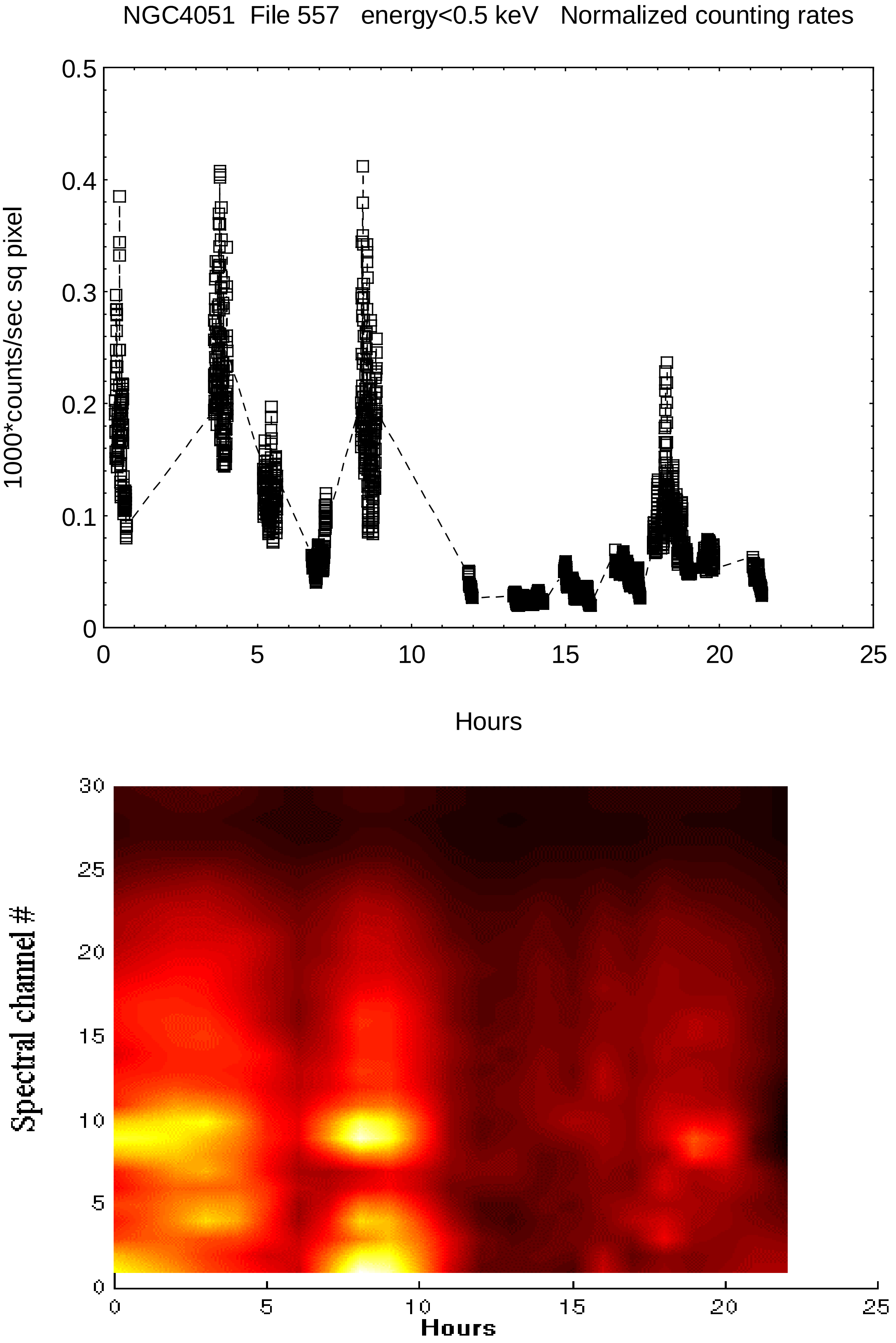


Fig. 7. NGC4051. Normalized measured counting rates for photons <0.5 keV (upper diagram) and the 3-D frequency spectrum of counting rate variations during the same sequence.

The wavelet analysis of short-term variations of the X-ray photon flux seems to open new possibilities to study properties of observed sources. In the present report the wavelet analysis was tested on two typical AGN. The same method will be applied to different types of X-ray sources.

Acknowledgement

The ROSAT data were accessed at the web address fttp://heasarc.gsfc.nasa.gov. The directory /rosat/doc/archive contains listings of available data.

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