Decomposition of particle energy spectra using the principal component method

Ludwik Liszka
Swedish Institute of Space Physics
Sörfors 634
S-905 88 Umeå
Sweden

When studying the particle energy spectra, particles with different energies are counted within the respective energy intervals. Also, in this case counting is performed during long time intervals so that large enough count numbers are obtained. A considerable fraction of temporal variations and thus the important information contained in them will be lost. For example, the variance pattern contains information about subpopulations present in the data. In order to preserve the information about the covariance of different spectral channels, high counting rates must be guaranteed by detector characteristics and not by long measuring intervals. In the present report a linear decomposition technique using the principal components is used to separate the subpopulations of particles present in the data. The described method will be applied to particle energy spectra measured onboard the Swedish satellite Viking (Hultqvist 1988).

The data used here contains electron and ion energy spectra between 0 and 40 keV collected by the satellite during the passage of the polar cusp region. In order to obtain a satisfactory number of energy spectra during a single satellite passage, the energy spectra were accumulated within each satellite spin interval. Only one population of energy spectra is studied in the present example: those collected at high pitch angles (PA>150 degrees i. e. particle detector looking downward along the geomagnetic field lines). The energy spectra used for the analysis contains 16 energy channels with a logarithmic energy scale in the interval 0 - 40 keV.

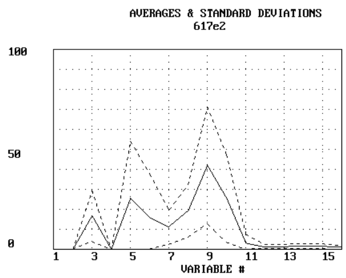
An example of the average counts for electrons and ions in that pitch angle interval is shown in Figure 1.

An interesting question is whether different parts of the energy spectrum are generated by the same process and if there are any relations between electrons and ions, which may be confirmed by the statistical methods. In order to test the above questions, the spectral data for both electrons and ions were combined in a single matrix, side by side, forming a 32 variable matrix.

It has been shown that multivariate time series, such as the energy spectra measured at equidistant time intervals, may be studied using the principal component method (Liszka, 1995). As the different generation mechanisms show in time as different variance patterns, the principal component method may resolve the entire particle energy spectrum into parts corresponding to the different generation mechanisms.

The method offers also a possibility to reconstruct the energy spectrum which would be observed if there would be only one generation mechanism at a time. The reconstruction procedure is as follows:

Electrons



Ions

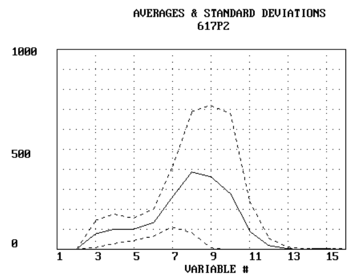


Fig. 1. An example of the average counts for electrons and ions in the pitch angle range Pa>150 (together with the ranges of double standard deviations. Orbit # 617.

A matrix **B**, consisting of N 32-component vectors (16 energy channels for electrons and 16 energy channels for ions), where N is number of satellite spins recorded during one orbit, is constructed for each satellite orbit.

The next step of the analysis was to perform the principal component analysis (PCA) for each satellite orbit. The results of PCA were:

- The vector of eigen values of the matrix, telling how much of the total variance in the matrix may be explained by the consecutive principal components.
- The matrix of component score coefficients **a**, a transformation matrix between the old system of 32 variables and the principal components (the new coordinate system).
- The matrix of component scores **S**, with one column for each principal component, being a projection of old 32 variables upon the new coordinate axis (directions of principal components).

Each principal component represents an independent mechanism controlling the variations of the 32-component vector, which instantaneous values are gathered in the matrix **B**.

If one wants to know what the variations of the 32-component vector would be with only one mechanism, corresponding to the principal component l active, it is possible to mask all other columns in **S**, except of column l and to perform a calculation of a new matrix B_l :

$$\mathbf{B}_l = \mathbf{S} \cdot \mathbf{a}^{-1}$$

The operation may be repeated for each interesting component l.

A disadvantage of the method is that a large amount of spectral vectors is needed to perform the analysis. In the present examples only orbits with the number of satellite spins larger than 50 were used.

The decomposition process has been performed for five Viking orbits: nos. 332, 392, 415, 469 and 618. The results are shown in Figs. 2 - 6. Each result displays the original combined electron-ion energy spectrum (electrons: channels 1-16, ions: channels 17-32). The spectra are plotted as three-dimensional colour coded graphs with a linear intensity scale. The number at the x-axis indicates the number of satellite spins during the recorded part of the orbit. Below the original spectrum partial spectra, each for one principal component, PC1 - PC4, are shown. The number beneath each partial spectrum indicates percentage of total variance explained by the respective principal component. Only significant principal components are displayed.

By definition each principal component is orthogonal to each other. It means that the partial spectra are constructed in such a way that each one depicts an independent part of the spectrum.

Some of the components (partial spectra) correspond to different particle populations. Even different kinds of instrumental effects may be identified among the principal components.

The structures observed in partial spectra are, in general, difficult to se in the original spectrum.

The present method may be used, in addition to identification of different particle populations, for removing instrumental effects from the data.

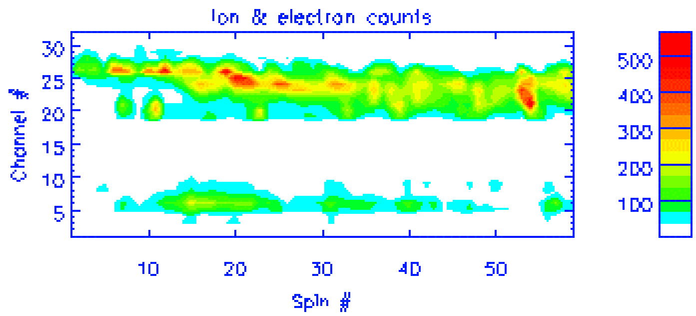
References

Hultqvist, B., Scientific Results from the Swedish Viking Satellite: A 1988 Status Report. In Scientific Report 196, Swedish Institute of Space Physics, 1988.

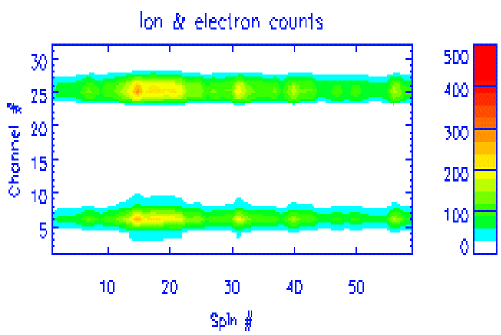
Liszka, L.: Causal modeling of spectral data: A new tool to study non-linear processes.

Scientific Report no.218, Swedish Institute of Space Physics, 1995.

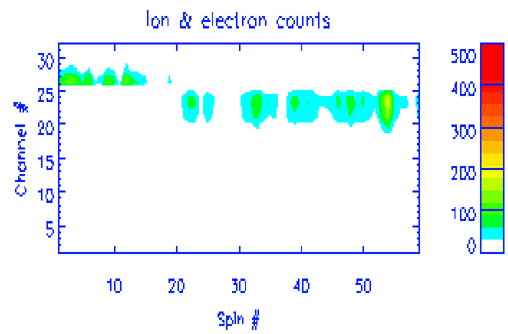
Orbit #332



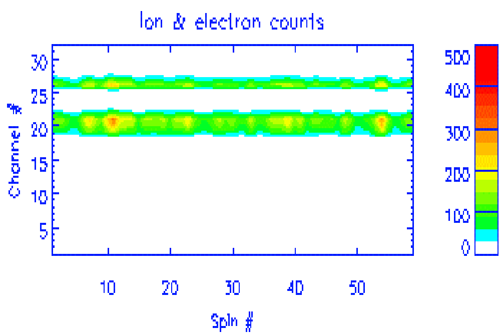
Original data



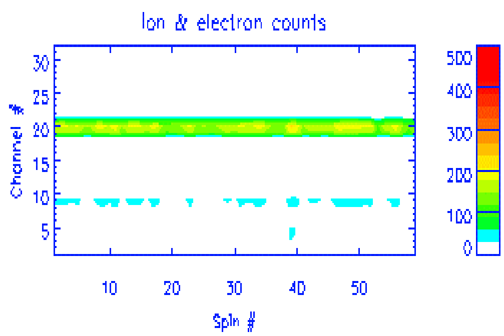
PC1 (32.7%)



PC2 (18.6%)



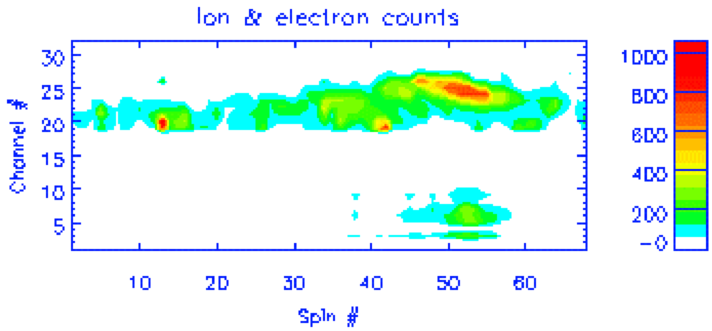
PC3 (8.0%)



PC4 (6.4%)

Fig. 2. Upper diagram: Particle energy spectra: electrons (channels 1 - 16) and ions (channels 17 - 32), pitch angles > 150 degrees, orbit #332. Lower diagrams: partial spectra corresponding most significant principal components.

Orbit #392



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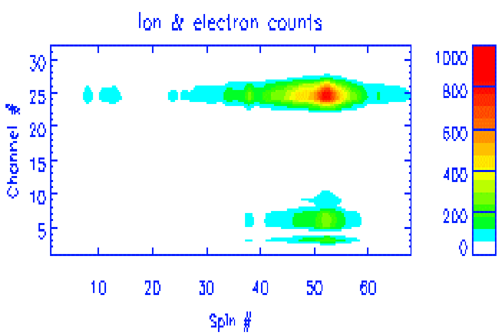
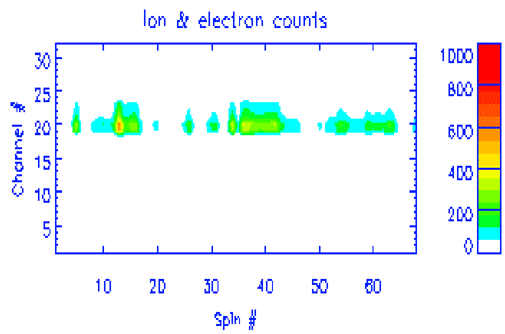
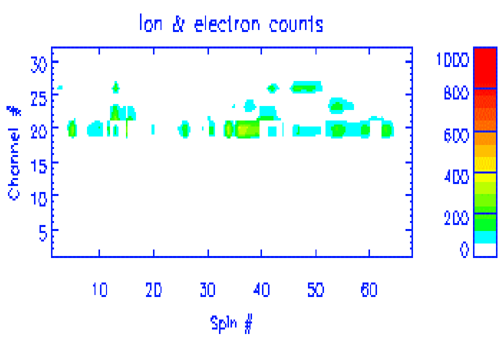


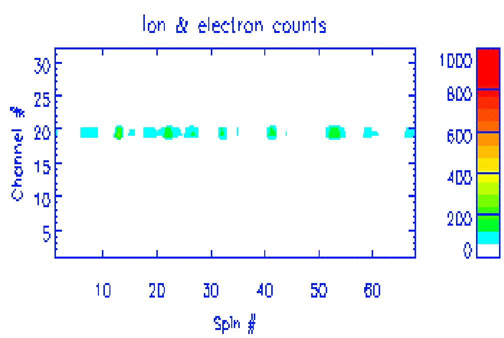
PC1 (50.6%)



PC2 (9.7%)



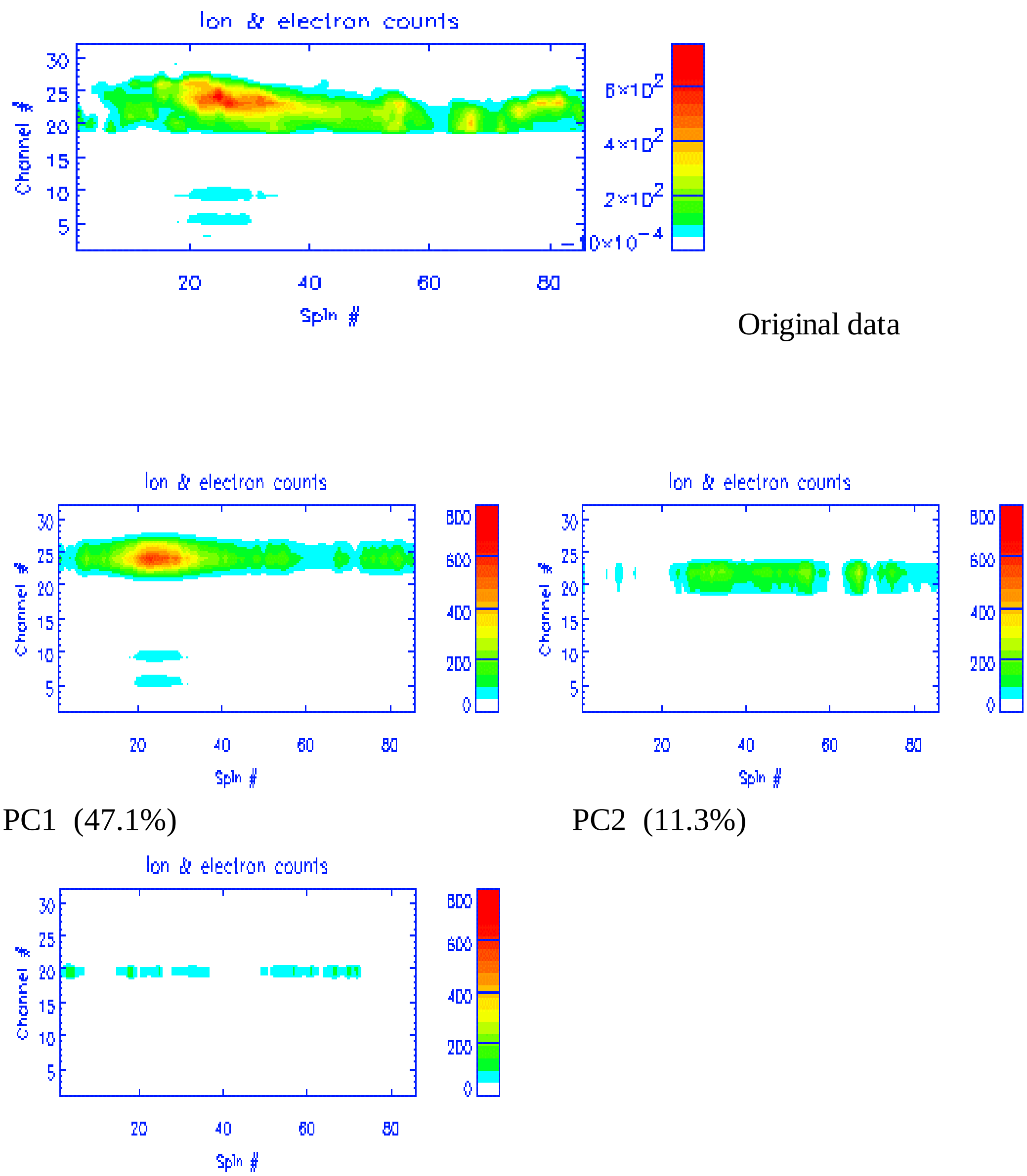
PC3 (6.3%)



PC4 (4.8%)

Fig. 3. Upper diagram: Particle energy spectra: electrons (channels 1 - 16) and ions (channels 17 - 32), pitch angles > 150 degrees, orbit #392. Lower diagrams: partial spectra corresponding most significant principal components.

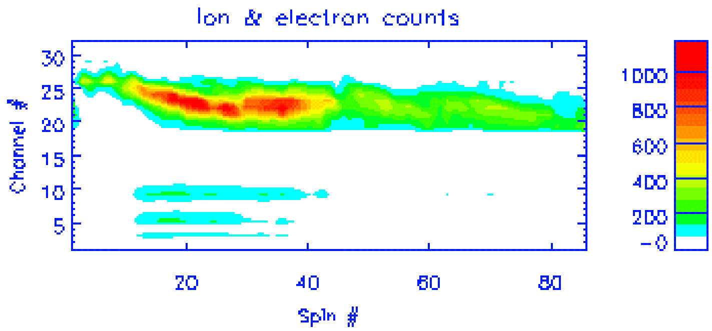
Orbit #415



PC3 (4.5%)

Fig. 4. Upper diagram: Particle energy spectra: electrons (channels 1 - 16) and ions (channels 17 - 32), pitch angles > 150 degrees, orbit #415. Lower diagrams: partial spectra corresponding most significant principal components.

Orbit #469



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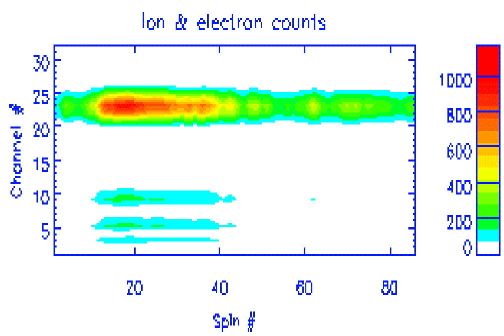
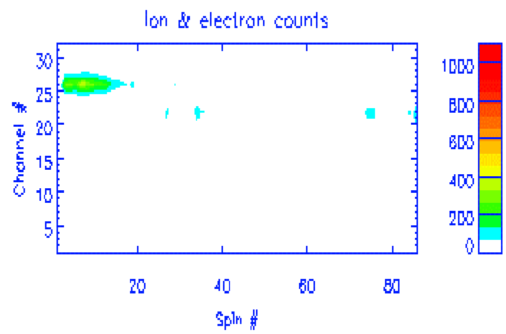
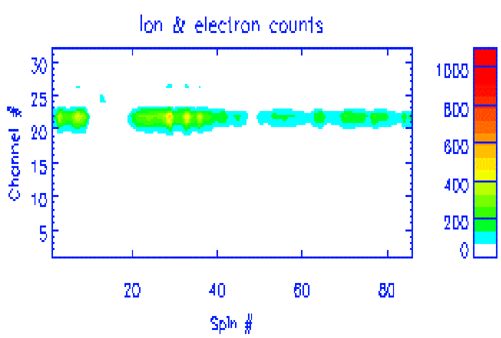




PC1 (38.5%)

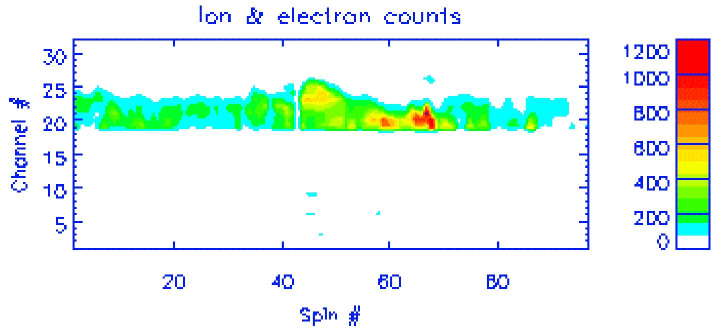
PC2 (19.9%)



PC3 (7.1%)

Fig. 6. Upper diagram: Particle energy spectra: electrons (channels 1 - 16) and ions (channels 17 - 32), pitch angles > 150 degrees, orbit #469. Lower diagrams: partial spectra corresponding most significant principal components.

Orbit #618



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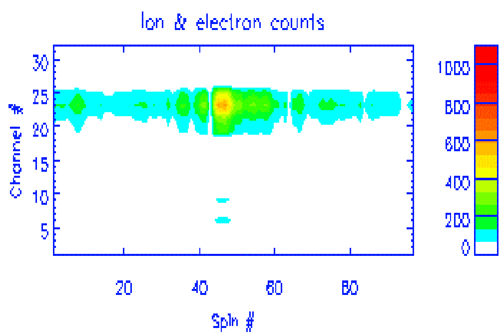
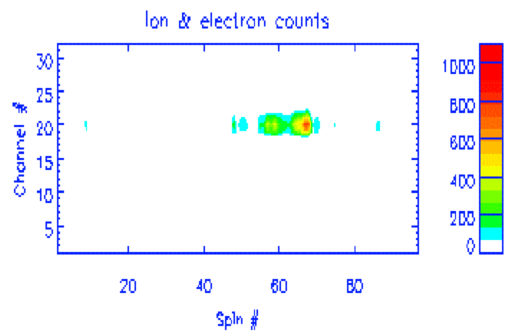
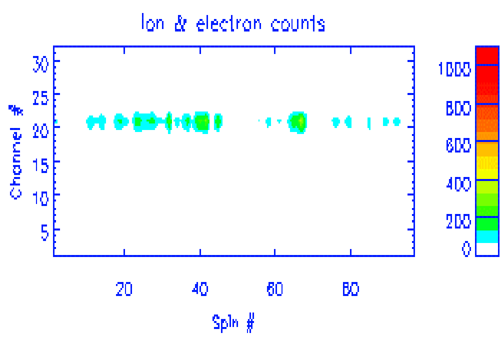


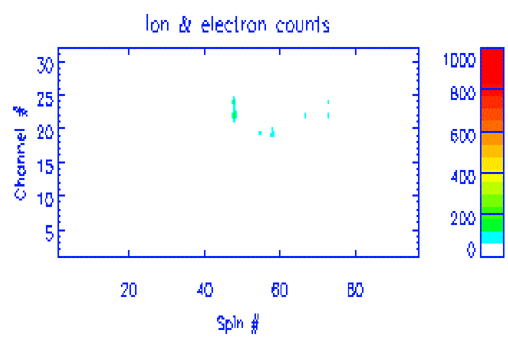
PC1 (38.3%)



PC2 (10.6%)



PC3 (7.7%)



PC4 (5.7%)

Fig. 7. Upper diagram: Particle energy spectra: electrons (channels 1 - 16) and ions (channels 17 - 32), pitch angles > 150 degrees, orbit #618. Lower diagrams: partial spectra corresponding most significant principal components.